MTH202 · DISCRETE MATHEMATICS

Lecture 1

Statements, logical connectives, translating English into symbols, and the first truth tables.

What you will learn

Step 1	What a statement (proposition) is
Step 2	Compound statements and the five logical connectives
Step 3	Translating English into symbols, and symbols back into English
Step 4	Truth tables for negation (~), conjunction (∧) and disjunction (∨)

Before we start: the course in one page

The first page of the handout introduces the course. In exams, the most useful part is the idea of **discrete** versus **continuous**.

Discrete	Continuous
Separate, individual points with gaps between them	No gaps; values flow smoothly
Example: the set of integers ..., -2, -1, 0, 1, 2, ...	Example: the set of real numbers

Discrete Mathematics concerns processes that consist of a sequence of individual steps.

Main topics of the course

- Logic
- Sets and operations on sets
- Relations and their properties
- Functions
- Sequences and series
- Recurrence relations
- Mathematical induction
- Loop invariants
- Combinatorics
- Probability
- Graphs and trees

Recommended books

- *Discrete Mathematics with Applications* (2nd edition) by Susanna S. Epp
- *Discrete Mathematics and Its Applications* (4th edition) by Kenneth H. Rosen
- *Discrete Mathematics* by Ross and Wright

Step 1: Statements (propositions)

Logic is the study of the principles and methods that distinguish between a valid and an invalid argument. Everything in logic is built from **statements**.

Key idea

A **statement** is a declarative sentence that is either **true** or **false**, but not both.

A statement is also called a **proposition**.

Its truth value is written **T** (true) or **F** (false).

Sentence	Statement?	Why
Grass is green.	Yes	It is true.
4 + 2 = 7	Yes	It is false, and a false sentence is still a statement.
-1 > 0	Yes	It is false.
Logic is interesting.	Yes	The handout counts it as a statement.
Close the door.	No	It is an order (command).
May I come in?	No	It is a question.
x + 2 is positive.	No	We do not know the value of x.
x + y = 12	No	We do not know x and y.
He is very rich.	No	We do not know who "he" is.

Memory trick

If the sentence is a **question**, an **order**, or has an **unknown** (x, y, he, she), it is **not** a statement.

The context rule

A sentence with a variable or a pronoun **becomes** a statement when earlier sentences make it clear.

- "x = 1. x > 2." Now x is known, so "x > 2" is a statement. Its truth value is **F**.
- "Bill Gates is an American. He is very rich." Now we know who "he" is, so "He is very rich" is a statement. Its truth value is **T**.

Practice

1. Lahore is in Pakistan. Is it a statement?
2. Please sit down. Is it a statement?
3. x + y = 12. Is it a statement?
4. 5 > 10. Is it a statement?

Answers (cover them first, then check)

1. Yes, a statement. Truth value T.
2. Not a statement. It is an order.
3. Not a statement. x and y are unknown.
4. Yes, a statement. Truth value F.

Step 2: Compound statements and connectives

When simple statements are joined with words such as **and**, **or**, **not**, the result is a **compound statement**. For example: "3 + 2 = 5" **and** "Lahore is a city in Pakistan".

Statements are written with small letters such as **p**, **q**, **r**.

The five connectives (learn this table by heart)

Connective	Meaning	Symbol	Name of symbol
Negation	not	~	Tilde
Conjunction	and	∧	Hat
Disjunction	or	∨	Vel
Conditional	if … then	→	Arrow
Biconditional	if and only if	↔	Double arrow

Memory trick

- ∧ is closed at the top and looks like the letter A, for **A**nd.
- ∨ is the opposite shape: an upside-down A, for or.
- **Con**junction = and. **Dis**junction = or.
- If asked "What is ∨ called?", the answer is **Vel**. If asked "Which connective is ∨?", the answer is **Disjunction**.

Example

Let p = "Islamabad is the capital of Pakistan" and q = "17 is divisible by 3".

- **p ∧ q**: Islamabad is the capital of Pakistan **and** 17 is divisible by 3.
- **p ∨ q**: Islamabad is the capital of Pakistan **or** 17 is divisible by 3.
- **~p**: It is not the case that Islamabad is the capital of Pakistan (or simply: Islamabad is **not** the capital of Pakistan).

A closer look: conditional (→)

p → q means "**if** p, **then** q". Think of it as a **promise**.

Let p = "You study hard" and q = "You will pass". The teacher promises: "If you study hard, then you will pass."

p (studied hard)	q (passed)	Was the promise broken?	p → q
T	T	No	**T**
T	F	Yes: you studied but failed	**F**
F	T	No: the teacher said nothing about not studying	**T**
F	F	No	**T**

Key idea

p → q is false in **only one** case: when p is **T** and q is **F**.

A closer look: biconditional (↔)

p ↔ q means "p **if and only if** q": the two always go together.

Let p = "The light is on" and q = "The switch is on". "The light is on if and only if the switch is on."

p (light)	q (switch)	Do they match?	p ↔ q
T	T	Yes, both on	**T**
T	F	No	**F**
F	T	No	**F**
F	F	Yes, both off	**T**

Key idea

p ↔ q is true when p and q have the **same** truth value (both T or both F).

→ is a one-way road: "if this, then that". ↔ is a two-way road: "this only when that".

Note

Lecture 1 only names → and ↔. Their full truth tables are studied in later lectures.

Practice

1. Which symbol is used for "or", and what is it called?
2. What does "conjunction" mean?
3. What is the name of the symbol ~ ?
4. Let p = "It is cold" and q = "It is raining". Write p ∧ q in English.
5. Let p = "It rains" and q = "The ground is wet". Write p → q in English.
6. When is p → q false?
7. If p = T and q = F, what is p ↔ q?

Answers (cover them first, then check)

1. ∨, called Vel (the connective is disjunction).
2. and.
3. Tilde.
4. It is cold and it is raining.
5. If it rains, then the ground is wet.
6. Only when p is true and q is false.
7. F, because p and q do not match.

Step 3: Translating between English and symbols

Three words cause most mistakes. Learn them first.

English word	Symbol	Why
not	~	Negation.
but	∧	"Ali is poor but happy" means Ali is poor **and** happy.
neither … nor	~p ∧ ~q	"Not this **and** not that".

English to symbols

Let p = "It is hot" and q = "It is sunny".

Sentence	Symbolic form
It is not hot.	**~p**
It is hot and sunny.	**p ∧ q**
It is hot or sunny.	**p ∨ q**
It is not hot but sunny.	**~p ∧ q**
It is neither hot nor sunny.	**~p ∧ ~q**

Example

Let h = "Zia is healthy", w = "Zia is wealthy", s = "Zia is wise".

- Zia is healthy and wealthy but not wise: **(h ∧ w) ∧ (~s)**
- Zia is not wealthy but he is healthy and wise: **~w ∧ (h ∧ s)**
- Zia is neither healthy, wealthy nor wise: **~h ∧ ~w ∧ ~s**

Symbols to English

Let m = "Ali is good in Mathematics" and c = "Ali is a Computer Science student".

Symbolic form	English
~c	Ali is not a Computer Science student.
c ∨ m	Ali is a Computer Science student or good in Maths.
m ∧ ~c	Ali is good in Maths but not a Computer Science student.

Memory trick

When ∧ is followed by **~**, the English "**but not**" usually sounds natural.

In the exam, draw the real symbol ∧ (a pointed A). Do not write the computer caret ^.

Practice

1. Let h = "Zia is healthy", s = "Zia is wise". Write "Zia is healthy but not wise" in symbols.
2. Let h = "Zia is healthy", w = "Zia is wealthy". Write "Zia is neither healthy nor wealthy" in symbols.
3. Let w = "Zia is wealthy", s = "Zia is wise". Write ~w ∧ s in English.
4. Let h = "Zia is healthy", s = "Zia is wise". Write h ∨ s in English.

Answers (cover them first, then check)

1. h ∧ ~s
2. ~h ∧ ~w
3. Zia is not wealthy but wise.
4. Zia is healthy or wise.

Step 4: Truth tables for ~, ∧ and ∨

Key idea

A **truth table** shows the truth value of a compound statement for **every possible** combination of truth values of its parts.

- One letter (p): **2 rows**. Two letters (p, q): **4 rows**.
- For two letters, always write the rows in this order: **TT, TF, FT, FF**.

Negation (~p): flip it

p	~p
T	**F**
F	**T**

Conjunction (p ∧ q): true only when both are true

p	q	p ∧ q
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**F**

> **Example**
> "I brought tea **and** biscuits." If even one of them is missing, the sentence is false.

Disjunction (p ∨ q), also called inclusive or: true when at least one is true

p	q	p ∨ q
T	T	**T**
T	F	**T**
F	T	**T**
F	F	**F**

> **Example**
> "I will bring tea **or** biscuits." If I bring either one (or both), the sentence is true. It is false only if I bring nothing.

> **Remember in one line each**
> - **~**: flip the value.
> - ∧: only **TT** gives T; everything else is F.
> - ∨: only **FF** gives F; everything else is T.

Practice

1. If p = F, what is ~p?
2. If p = T and q = F, what is p ∧ q?
3. If p = T and q = F, what is p ∨ q?
4. If p = F and q = F, what is p ∨ q?
5. If p = T and q = T, what is ~p ∧ q? (Hint: first find ~p, then apply ∧.)

Answers (cover them first, then check)

1. T
2. F
3. T
4. F (∨ is false only when both are false)
5. F (~p = F, and F ∧ T = F)

Revision sheet: Lecture 1

Topic	What to remember
Statement	True or false, not both. Questions, orders and unknowns (x, he) are not statements.
Context rule	Earlier sentences can make x or "he" clear; then it becomes a statement.
Negation	not, ~, Tilde
Conjunction	and, ∧, Hat
Disjunction	or, ∨, Vel
Conditional	if … then, →, Arrow. False only when T → F.
Biconditional	if and only if, ↔, Double arrow. True when both values match.
Tricky words	but = ∧; neither p nor q = ~p ∧ ~q
~ table	Flip the value.
∧ table	Only TT is T.
∨ table	Only FF is F.
Discrete vs continuous	Integers are discrete (gaps); real numbers are continuous (no gaps).